Math 347: Lecture 2 - Worksheet

August 29, 2018

1) What are the domain and image of the absolute value function?

2) Let $A = \{\text{January,February}, \ldots, \text{December}\}$. Given $x \in A$, let $f(x)$ be the number of days in x. Does f define a function from A to $\mathbb{N}$? What is its domain and range?

3) Define the image of the functions $f : \mathbb{R} \to \mathbb{R}$ defined below:

   a. $f(x) = \frac{x^2}{1+x^2}$;

   b. $f(x) = \frac{x}{|1+x|}$.

4) Let $f : \mathbb{N} \times \mathbb{N} \to \mathbb{R}$ be defined by $f(a, b) = \frac{(a+1)(a+2b)}{2}$.

   a. Show that the image of f is contained in $\mathbb{N}$.

   b. Determine exactly which natural numbers are in the image of f.

5) For S in the domain of a function f, let $f(S) = \{f(x) \mid x \in S\}$. Let C and D be subsets of the domain of f.

   a. Prove that $f(C \cap D) \subset f(C) \cap f(D)$.

   b. Give an example where the equality doesn't hold in part a.

6) When $f : A \to B$ and $S \subset B$, we define $I_f(S) = \{x \in A \mid f(x) \in S\}$. Let X and Y be subsets of B:

   a. Determine whether $I_f(X \cup Y)$ is equal to $I_f(X) \cup I_f(Y)$.

   b. Determine whether $I_f(X \cap Y)$ is equal to $I_f(X) \cap I_f(Y)$.

7) Let $S = \{x \in \mathbb{R} \mid x(x-1)(x-2)(x-3) < 0\}$. Let T be the interval $(0, 1)$ and U be the interval $(2, 3)$. Determine the relations between the sets S, T and U.

8) Let $S = [3] \times [3]$ (the Cartesian product of $\{1, 2, 3\}$ with itself). Let T be the set of ordered pairs $(x, y) \in \mathbb{Z} \times \mathbb{Z}$ such that $0 \leq 3x + y - 4 \leq 8$. Prove that $S \subset T$. Does equality hold?